An Introduction to Generalized Global Symmetry

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June 2026

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Ordinary Symmetry: Localizing the Parameter

For an ordinary continuous symmetry with a constant parameter ϵ, We derive the Noether current by making it local:

$$\epsilon \longrightarrow \epsilon(x).$$

Then the variation of the action has the form

$$\delta S = \int_M d^n x \, J^\mu \partial_\mu \epsilon = -\int_M \mathrm{d}\epsilon \wedge j = \int_M \epsilon \, \mathrm{d}j.$$

Here the current form j is

$$j_{\mu_1 \ldots \mu_{n-1}} = \epsilon_{\mu_1 \ldots \mu_n} J^{\mu_n}$$

is the **current form** of degree $n-1$ dual to the vector current J^μ ($n = \dim M$).

Ordinary Global Symmetry Condition

The action S is invariant when ϵ is a constant. In differential-form language:

$$\mathrm{d}\epsilon = 0.$$

When the equations of motion are satisfied, $\delta S = 0$ implies the current conservation:

$$\mathrm{d}j = 0.$$

(same as $\partial_\mu J^\mu = 0$).

The symmetry charge $Q(\Sigma)$ is defined on codimension 1 surface Σ as:

$$Q(\Sigma) = \int_\Sigma j.$$

Ordinary Symmetry as a Surface Operator

Because $\mathrm{d}j = 0$, the charge is topological under deformations of Σ:

$$\Sigma \sim \Sigma' \qquad \Longrightarrow \qquad Q(\Sigma) = Q(\Sigma')$$

When it doesn't cross charged operators.

In the presence of charged operators, we have **Ward Identities**:

$$\langle Q(\Sigma) O(x) \rangle = L(\Sigma, x) \langle \delta O(x) \rangle.$$

The **linking number** $L(\Sigma, x) = 0, 1$ depends on whether Σ surrounds x. For finite transformation:

$$\langle U(g, \Sigma) O(x) \rangle = R(g)^{L(\Sigma, x)} \langle O(x) \rangle$$

The Key Generalization

Generalization required for p form symmetry:

Replace the scalar parameter ϵ by a p-form-valued parameter.

Variation of the action:

$$\delta S = -\int_{\mathcal{M}} \mathrm{d}\epsilon \wedge j = \pm \int_{\mathcal{M}} \epsilon \wedge \mathrm{d}j.$$

Global symmetry condition:

$$\mathrm{d}\epsilon = 0.$$

For classical solutions, $\delta S = 0$ leads to current conservation:

$$dj = 0$$

Current Form and Charge for a p-Form Symmetry

For p-form symmetry, current forms j has degree $d - p - 1$.

Charge $Q(\Sigma)$ is defined on closed submanifold of dimension $d - p - 1$.

$$\Sigma^{d-p-1} \subset \mathcal{M}.$$

Equivalently, Σ has codimension $p + 1$.

The Noether charge remains the same:

$$Q(\Sigma) = \int_{\Sigma} j.$$

Charged Operators and Linking

A p-form symmetry acts on p-dimensional charged operators

$$\mathcal{O}(C), \qquad \dim C = p.$$

The symmetry operator lives on

$$\dim \Sigma = d - p - 1.$$

Therefore

$$\dim C + \dim \Sigma = d - 1,$$

so C and Σ can have a **linking number**

$$L(\Sigma, C).$$

The symmetry acts only when Σ links (surrounds) the charged operator on C.

General Definition: p-Form Global Symmetry

More generally, a p-form global symmetry has symmetry operators

$$U(g, \Sigma)$$

supported on closed codimension-$(p+1)$ submanifolds Σ.

Action on p-dimensional charged operators $\mathcal{O}(C)$ surrounded by Σ:

$$\langle U(g, \Sigma)\mathcal{O}(C)\rangle = R(g)^{L(\Sigma, C)}\langle \mathcal{O}(C)\rangle.$$

Satisfies Group multiplication law:

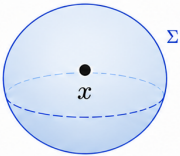
$$U(g, \Sigma)U(h, \Sigma) = U(gh, \Sigma)$$

Illustration: Higher Form Symmetry

0-form vs 1-form Symmetry

0-form (ordinary) symmetry

Symmetry operator:
$U_0(g, \Sigma)$ on a closed $(d-1)$-dimensional surface Σ



point operator $O(x)$

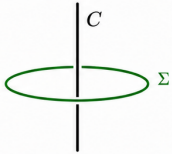
symmetry surface Σ

Action: $U_0(g, \Sigma)\, O(x) = R_0(g)\, O(x)$
if x is inside Σ
(trivial if x is outside Σ)

Charged objects: local (0-dimensional) operators
They live **at points**.

1-form symmetry

Symmetry operator:
$U_1(g, \Sigma)$ on a closed $(d-2)$-dimensional submanifold Σ



line operator $O(C)$

symmetry loop Σ
(closed codimension-2 submanifold)

Action: $U_1(g, \Sigma)\, O(C) = R_1(g)^{L(\Sigma, C)}\, O(C)$
where $L(\Sigma, C)$ is the linking number
(# of times C links with Σ)

Charged objects: line-like (1-dimensional) operators
They live **along curves**.

General rule (p-form symmetry in d dimensions):

- Symmetry operator $U_p(g, \Sigma^{d-p-1})$ on a closed codimension $(p+1)$ surface Σ
- Charged operator $O(C^p)$ on a p-dimensional submanifold C

$$U_p(g, \Sigma)\, O(C) = R_p(g)^{L(\Sigma, C)}\, O(C)$$
with $L(\Sigma, C)$ the linking number

Two Important Comments

1. Continuous vs. discrete symmetries.

For continuous symmetries, conserved currents motivate the construction:

$$\mathrm{d}j = 0, \qquad U(\alpha, \Sigma) = \exp\left(i\alpha \int_{\Sigma} j\right).$$

Discrete symmetries have no current, but the topological operator definition still work.

2. Higher-form symmetries are Abelian.

For $p > 0$, there is no invariant notion of "inside" and "outside" for surfaces Σ^{d-p-1}, thus the symmetry group is Abelian.

Example: 4D Free Maxwell Theory

Consider Maxwell theory without dynamical electric charges:

$$S = \frac{1}{2e^2} \int F \wedge \star F.$$

There are two natural conserved 2-form currents (Magnetic/Electric Charge):

$$j_m = \frac{F}{2\pi}, \qquad j_e = \frac{\star F}{e^2}.$$

Their conservation follows from

$$\mathrm{d}F = 0 \qquad \text{and} \qquad \mathrm{d} \star F = 0.$$

Since these currents are 2-forms in $d = 4$, they generate **1-form symmetries**.

Electric One-Form Symmetry

Electric charge operator is supported on closed 2-surfaces Σ:

$$Q_e(\Sigma) = \int_\Sigma j_e = \frac{1}{e^2} \int_\Sigma \star F.$$

The charged operator is the Wilson loop/Electron world-line.

$$W_e(C) = \exp\left(i \int_C A \right).$$

$Q_e(\Sigma) = n$ counts the number of electrons enclosed by Σ, which is the linking number $L(\Sigma, C)$.

$$U_e(\Sigma, \alpha) W_e(C) = \exp\left(i\alpha L(\Sigma, C) \right) W_e(C).$$

Magnetic One-Form Symmetry

Similarly, the magnetic charge operator is

$$Q_m(\Sigma) = \int_\Sigma j_m = \frac{1}{2\pi} \int_\Sigma F.$$

Naively, since $F = \mathrm{d}A$, this looks trivial on closed surfaces.

But when the $U(1)$ bundle has nontrivial topology, A is only locally defined. Then

$$\frac{1}{2\pi} \int_\Sigma F$$

measures the magnetic flux through Σ. The charged objects are magnetic monopole worldlines ('t Hooft lines), and $Q_m(\Sigma) = n$ counts the number of magnetic monopoles inside Σ.

Spontaneous Breaking: Area vs. Perimeter Law

For one-form symmetry, large Wilson loops plays the role of long-range correlators of charged operators.

$$\langle W(C)\rangle \sim \langle O(x)O(y)\rangle \tag{1}$$

There are two possible large-loop behaviors:

$$-\log\langle W(C)\rangle \sim \begin{cases} \mathrm{Area}(C), & \text{unbroken / confined,} \\ \mathrm{Perimeter}(C), & \text{broken / deconfined.} \end{cases}$$

Area law means separated charges cost energy growing with distance.
Perimeter law means long-range order for extended operators.

Physical Interpretation of the Area Law

Take a rectangular Wilson loop with time length β and spatial separation $|x|$.

If the area law holds,

$$\langle W(C) \rangle \sim e^{-\beta |x|}.$$

This means the energy cost of separating charged particles grows linearly:

$$\Delta E \sim |x|.$$

So isolated charged particles do not appear in the infrared theory.

This is the confinement interpretation of unbroken one-form symmetry.

Photon as a Goldstone Boson

For ordinary symmetry breaking, coupling to a background gauge field gives

$$(A + \mathrm{d}\phi)^2,$$

so ϕ is the Goldstone mode.

For a broken p-form symmetry, couple to a background $(p+1)$-form field B:

$$S_{\text{eff}} \sim \int (B + \mathrm{d}a) \wedge \star(B + \mathrm{d}a).$$

Setting $B = 0$ gives

$$S_{\text{eff}} \sim \int \mathrm{d}a \wedge \star \mathrm{d}a.$$

Photon could be the Goldstone mode of a broken 1-form symmetry!

Takeaways

1. Ordinary symmetry can be described by topological surface operators.
2. The continuous motivation generalizes from

$$\delta S = \int \epsilon \, \mathrm{d}j \quad \text{to} \quad \delta S = \int \epsilon \wedge \mathrm{d}j.$$

3. For a p-form symmetry, the parameter is a closed p-form: $\mathrm{d}\epsilon = 0$.
4. Charged objects are p-dimensional, and the action is controlled by linking.

References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