Instantons in Superinductors

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Physics 332

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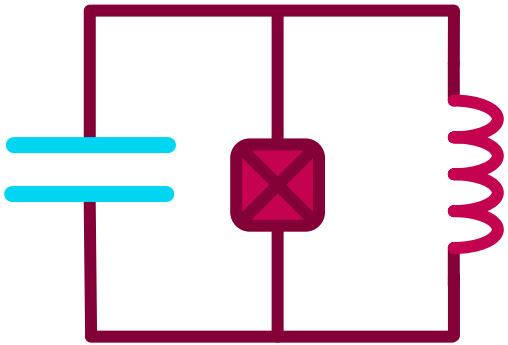
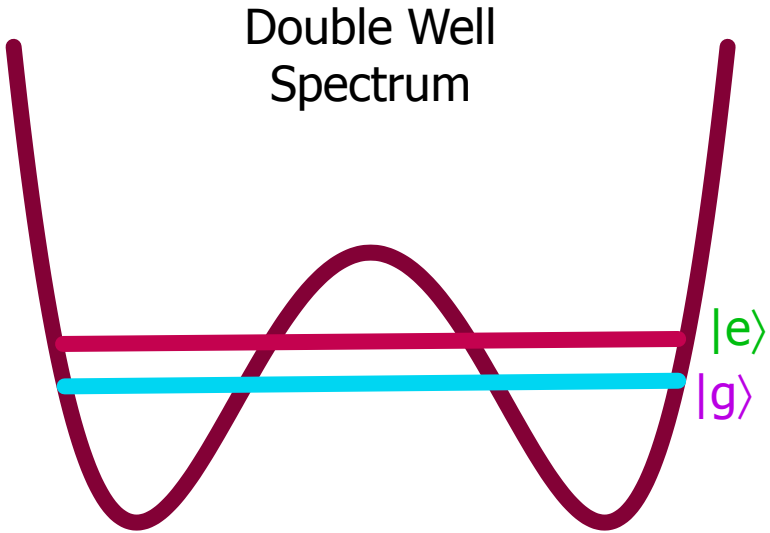
Outline

- Some brief background and motivation
  - Superconducting circuits and applications to Quantum Information
  - How superinductors fit into that picture & implementation
- Josephson Junctions
  - Josephson Potential
  - Role of Instantons in a Single Junction
- 1D Josephson Junction Array
  - Lagrangian & Stationary Solutions
  - Role of Instantons & higher order processes
  - Aharnov-Casher Effect
- Dephasing in Fluxonium due to the superinductor

Background



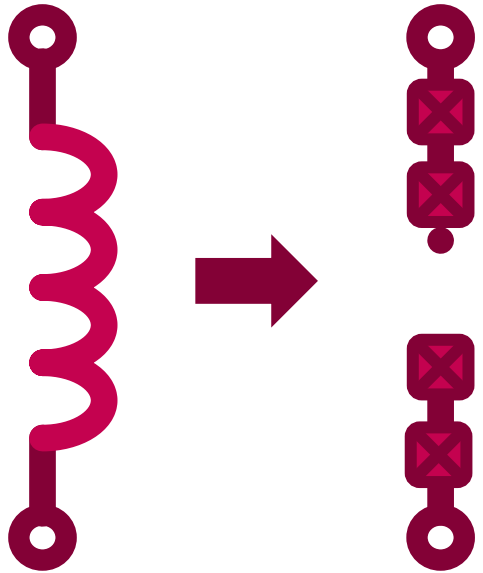
Fluxonium Circuit



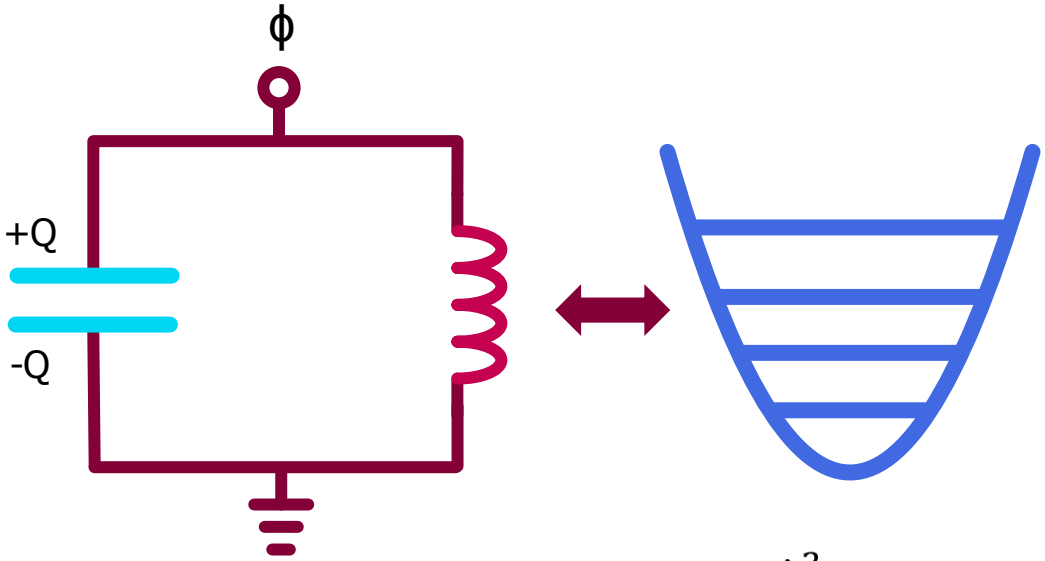
Gives rise to a naturally protected transition

Background (Cont.)

- How to implement the Inductor?
- Challenges
  - These circuits operate at microwave frequencies (high L is hard)
  - You need a large inductor to make this double well deep
- Solution
  - Implement the inductor using a Junction Chain



Circuit Quantization



$$\mathcal{L} = \frac{1}{2}C\dot{\phi}^2 + \frac{\phi^2}{2L}$$
$$[\phi, Q] = i\hbar$$

$$\mathcal{L} = \frac{\dot{\varphi}^2}{16E_C} + E_L\varphi^2$$
$$[\varphi, n] = -i\hbar$$

Classical Variable	Circuit Variable	Reduced Variable
Position (x)	Flux (ϕ) $\int V\,dt$	Mag. Flux
Momentum (p)	Charge (Q)	CP # (n)
Mass m	Capacitance C	E_C
Inv. Spring (k)	Inductance (L)	E_L

Josephson Potential

- Start with a tight-binding model

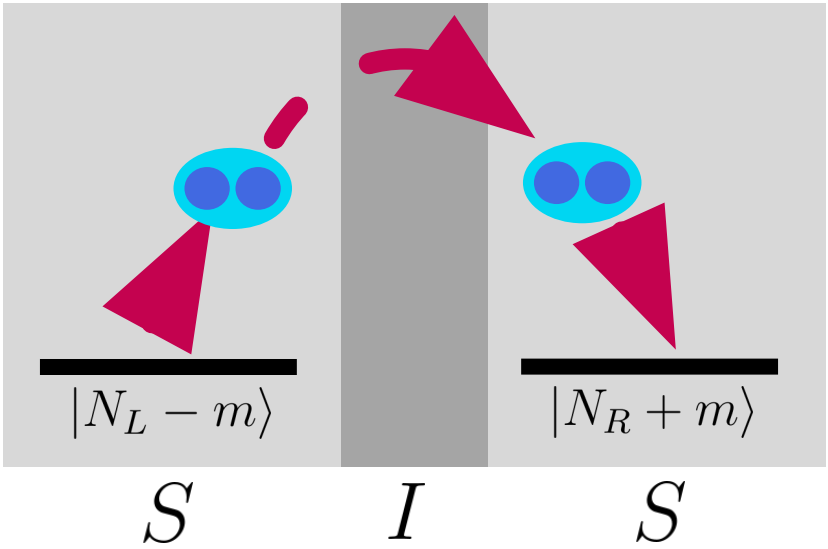
$$H = -\frac{E_J}{2}\sum_m |m\rangle\langle m+1| + h.c.$$

- Can transform into (φ) basis

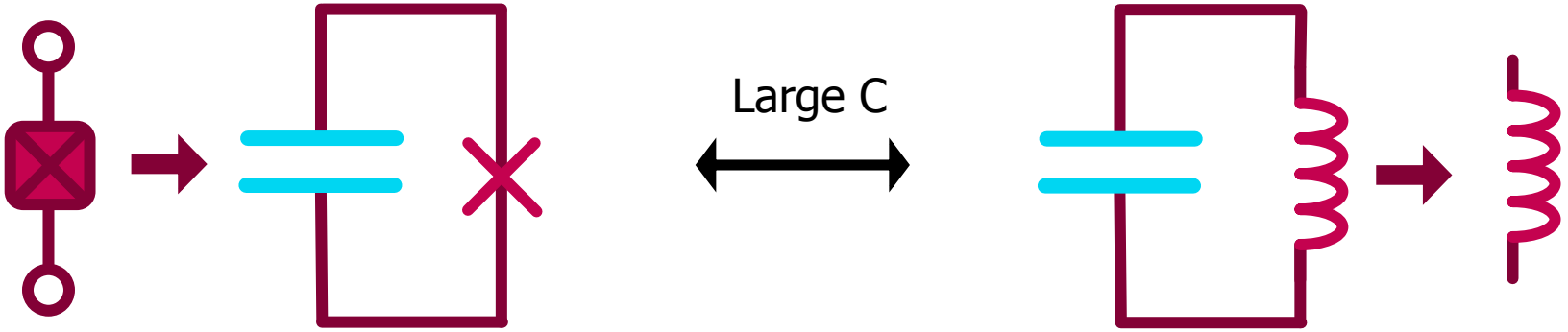
$$H = -E_J \cos(\varphi)\,|\varphi\rangle$$

- Make the potential positive

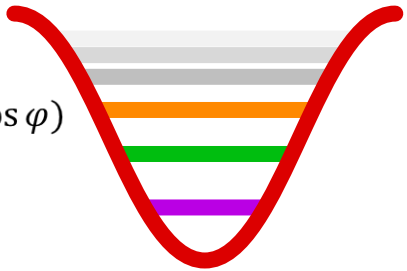
$$V(\varphi) = E_J\,(1-\cos\varphi)$$



Josephson Potential (Cont.)



$$\mathcal{L} = \frac{\dot{\varphi}^2}{16E_c} + E_J(1 - \cos\varphi)$$





$$\mathcal{L} = \frac{\dot{\varphi}^2}{16E_c} + \frac{E_J}{2}\varphi^2$$

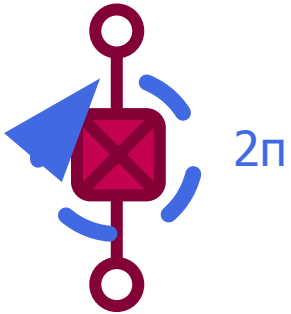
Single Junction Instantons

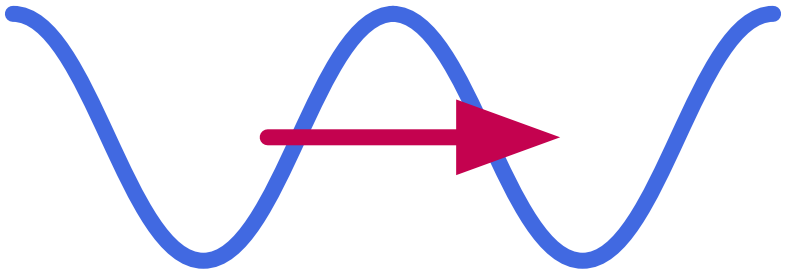
- In a single junction an instanton takes us from one Josephson well to another

$$S_I = \int_0^{2\pi} d\varphi \sqrt{2mV(\varphi)} = \sqrt{\frac{8E_J}{E_C}}$$

- This instanton event is often called a phase slip, and its amplitude is

$$\varepsilon_{ps} = Ae^{-S_I}$$





Modelling a 1D Array



$$\mathcal{L} = \sum_j \frac{\dot{\varphi}_j^{\,2}}{16E_c} + E_J(1-\cos\varphi_j) + \left(\frac{\Phi_0}{2\pi}\right)^2 \sum_j \dot{\varphi}_j \sum_{i=j}^{N} q_i$$

$$\varphi_{ext} = \sum_j \varphi_j$$

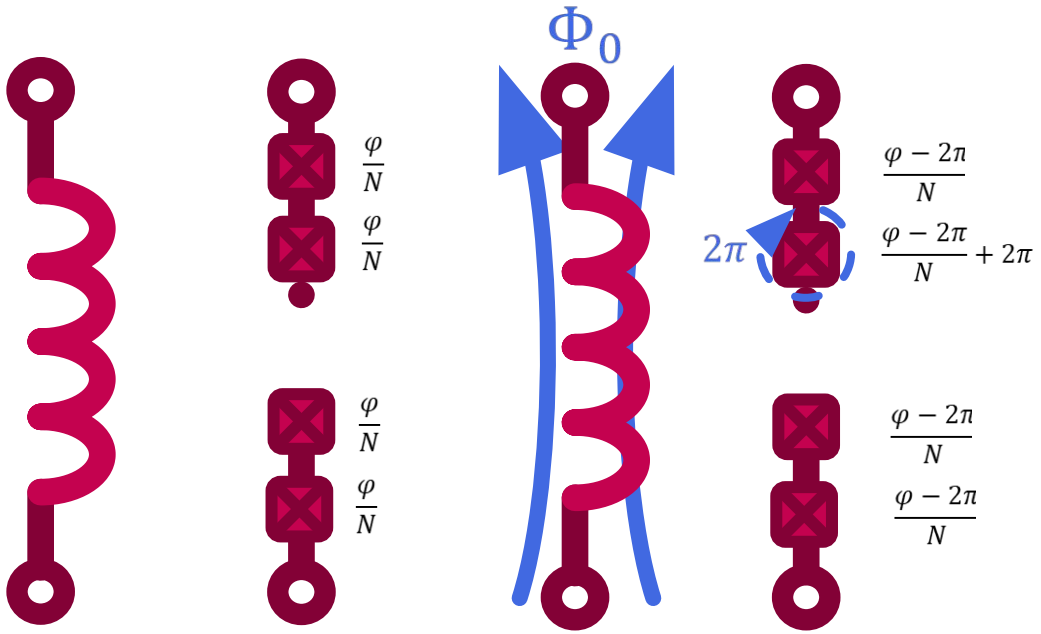
From Each Junction

Stray charges on each s.c. island

Constraint imposed by external bias

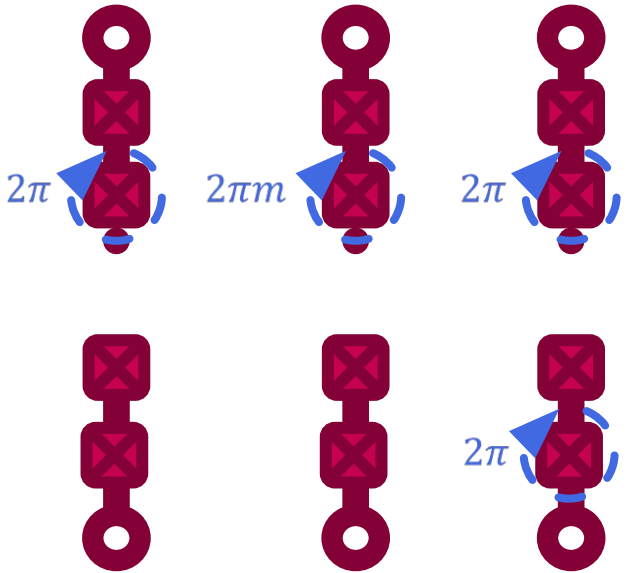
Stationary Solutions

- If we look for solutions where the phase doesn't change they are minima
- Many such solutions where one junction has slipped



Types of Instanton Events

- We will work in a regime where N is large
- There are three types of amplitudes to consider
  - A single junction in the array slips
  - A single junction slips many m-times
  - M junctions slip at the same time
- Only the first one will turn out to matter



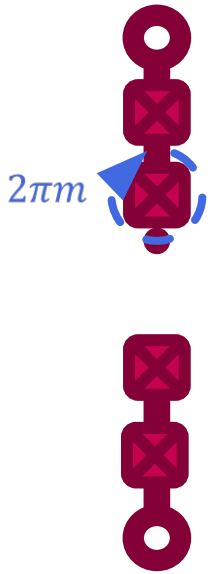
Correlated Phase Slips

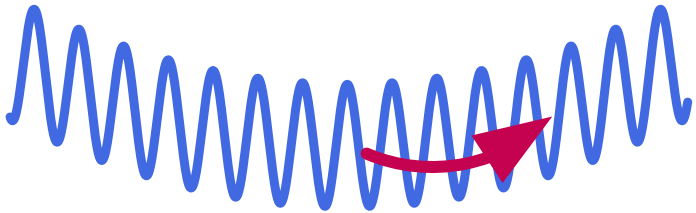
- The action goes like:

$$S_I \propto \int_0^{2\pi m} d\varphi \sqrt{1 - \cos\varphi + \frac{(\varphi_{ext} - \varphi)^2}{2N}}$$

- The relevant part is near the crest of the well, assuming this we can Taylor expand for large N to obtain

$$S_I \approx m\sqrt{\frac{8E_J}{E_C}} + Cm^3$$

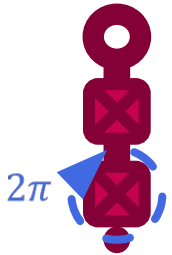




Many Junction Phase Slips

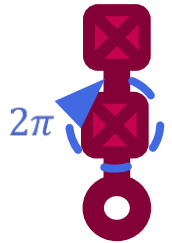
- The action goes like:

$$S_I = C\sqrt{m}\int_0^{2\pi} d\varphi \sqrt{m(1-\cos\chi) + \frac{(\varphi_{ext} - m\chi)^2}{2N}}$$



- When N is large the action can be Taylor expanded to show that

$$S_I \approx m\sqrt{\frac{8E_J}{E_C}} + Cm^2$$

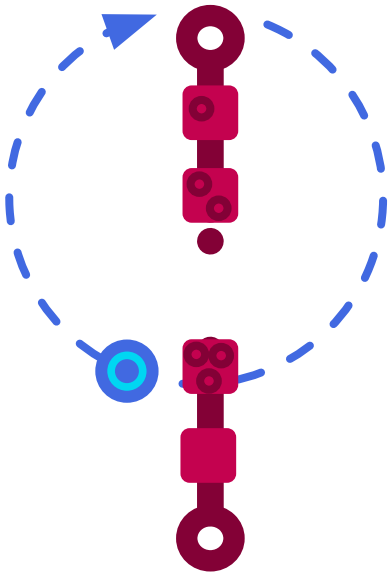


Aharonov-Casher Effect

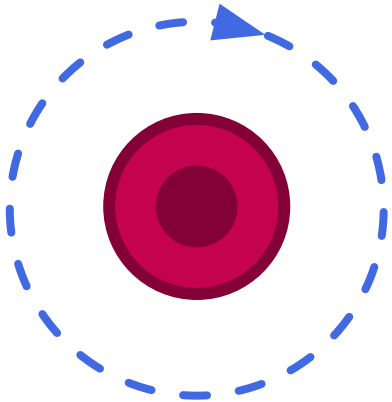
- Offset charge term gives a phase factor

$$S_{geo} = i \sum_{i=j}^{N} 2\pi n_i$$

- This phase factor is the manifestation of the Aharnov-Casher Effect



Island Picture



Aharonov-Bohm Phase

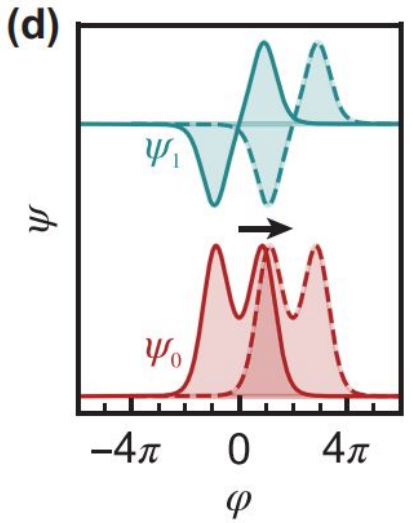
Phase Slip Hamiltonian Dephasing

- The Phase Slips in the inductor modify the Hamiltonian by adding a term to the Hamiltonian that looks like:

$$\delta H = \sum_j \frac{\varepsilon_{ps}^j}{2} e^{i \sum_{i=j}^{N} 2\pi n_i} |m+1\rangle\langle m| + h.c.$$

- Doing first order perturbation theory for the energy shift we find that

$$h\delta f_{01} = Re[\mathcal{F}_{01} E_{ps}]$$



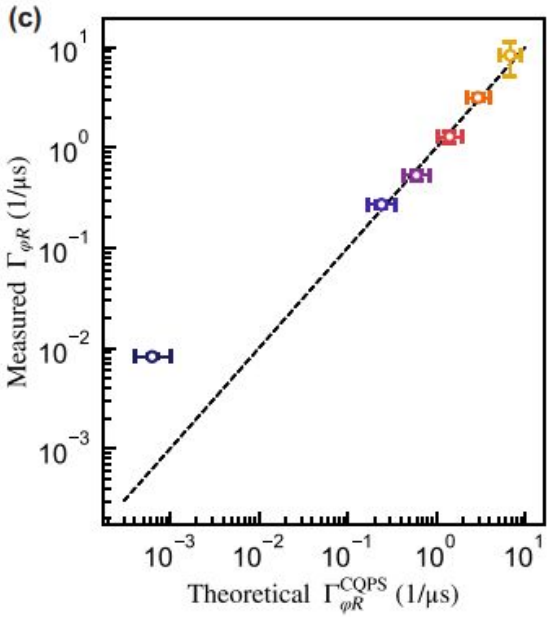
Note $\mathrm{E}_{ps} = \sum_j \frac{\varepsilon_{ps}^j}{2} e^{i \sum_{i=j}^{N} 2\pi n_i}$

https://journals.aps.org/prxquantum/pdf/10.1103/PRXQuantum.5.030341

Computing a Dephasing Rate

- Under normal bias conditions $\mathcal{F}_{01}$ is purely real so computing the variance of E_{ps} using the CLT we can get the spread of qubit energy values
- Converting this into a dephasing rate we find that

$$\Gamma_{Ps} = \pi\sqrt{N}\,\varepsilon_{ps}|\mathcal{F}_{01}|$$



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Thank You

Questions

Conclusion