Scaling the Sphaleron: Finite Temperature Baryogenesis in the Electroweak Theory

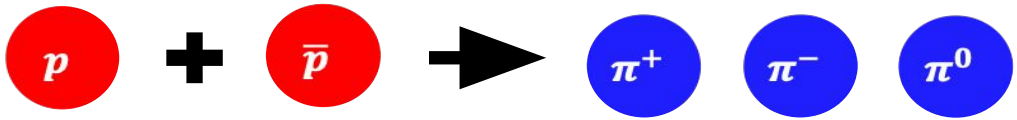
Chris Pezanosky

Stanford University

PHYS 332- Quantum Field Theory III

If physics is symmetric between particle and antiparticle, why does our universe overwhelmingly prefer matter?

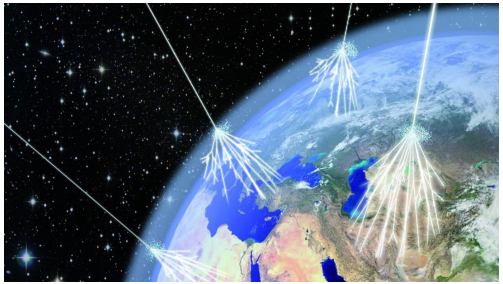
How do we know this?

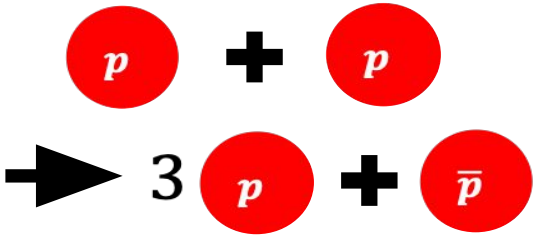


(atoms are pretty stable)

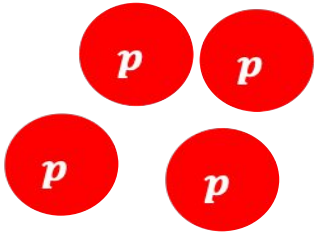
What about elsewhere in galaxy?

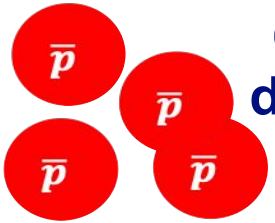
(cosmic rays)





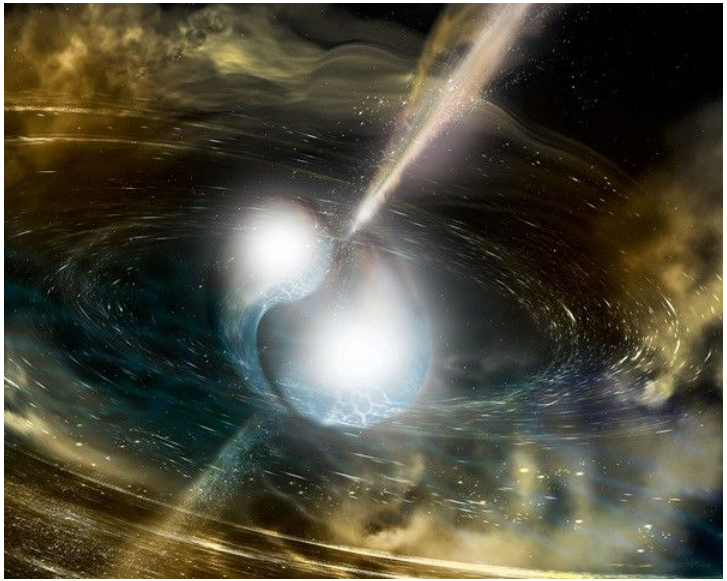
What about elsewhere in universe?





(no CMB distortion)

How *precisely* do we know the universe is asymmetric?



Nucleosynthesis models predict *all* light element abundances from 1 parameter:

$$\eta \equiv \frac{n_B}{s} = \frac{n_b - n_{\bar{b}}}{s}$$

Observation requires

$$1.5 \cdot 10^{-10} \leq \eta \leq 7 \cdot 10^{-10}$$

Why??

Explaining this *one* number is the goal of baryogenesis

Requires some way to violate 'baryon number' $B \equiv N_b - N_{\bar{b}}$

Electroweak Sector?

Vacuum Structure of Electroweak Theory

Electroweak Lagrangian (SU(2) + Higgs)

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(D_\mu \Phi)^\dagger D^\mu \Phi - \frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \frac{\lambda}{4}\left[\mathrm{Tr}(\Phi^\dagger \Phi) - v^2\right]^2$$

Look at Higgs...

(SU(2)-valued field)

$$\Phi^\dagger \Phi = (\phi_1^* \phi_1 + \phi_2^* \phi_2) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \Rightarrow \quad \Phi = \frac{\sigma}{\sqrt{2}} U \qquad U \in SU(2)$$

Vacuum State:

$$\Phi = \frac{v}{\sqrt{2}} U \qquad A_j = \frac{1}{ig} \partial_j U U^\dagger$$

(Sits at VEV) **(Pure gauge)**

$$U^\infty = \mathbb{I}$$

(Identify points at infinity)

$SU(2)$

S^3

EW vacuum defined by map $U(\mathbf{x}, t) : S^3 \mapsto SU(2)$

Winding Number!

Infinite spectrum of degenerate vacua characterized by Higgs winding

N_H 1 2 3 4

$$N_H(t) = \frac{1}{24\pi^2}\int d^3x \varepsilon^{ijk}\mathrm{Tr}[U^\dagger \partial_i U U^\dagger \partial_j U U^\dagger \partial_k U]$$

(Equal to gauge field winding in vacuum)

Can also define 'Chern-Simons number'

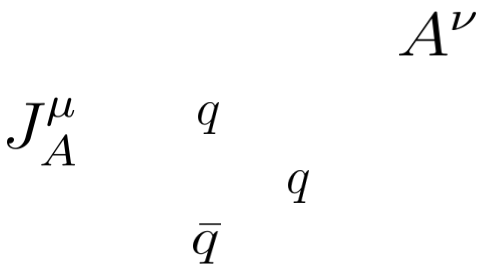
$$N_{CS}(t) = \int d^3x K^0_{CS} = \frac{g^2}{32\pi^2}\int d^3x \varepsilon^{ijk}\mathrm{Tr}\left(A_i\partial_j A_k + \frac{2}{3}igA_iA_jA_k\right)$$

Plugging in 'pure gauge' field from before, find

$$N_H = N_{CS}$$

Vacuum states *also* characterized by Chern-Simons number!

How does this relate to Baryon Number?



Define 'axial current'

$$J^\mu_A \equiv \frac{1}{3}\bar{q}\gamma^\mu\gamma^5 q = \frac{1}{3}(\bar{q}_L\gamma^\mu\gamma^5 q_L - \bar{q}_R\gamma^\mu\gamma^5 q_R)$$

Always **anomalous for gauge-coupled Dirac fermion**

What about *vectorlike* current?

$$J^\mu_B \equiv \frac{1}{3}\bar{q}\gamma^\mu q = \frac{1}{3}(\bar{q}_L\gamma^\mu q_L + \bar{q}_R\gamma^\mu q_R)$$

When LH and RH fields couple the same (e.g. QED), contributions to vector anomaly cancel

This is *not* the case for Electroweak Sector- SU(2) doesn't even talk to RH fields!

$$\partial_\mu J^\mu_B = n_f\left(\frac{g^2}{32\pi^2}W^a_{\mu\nu}\tilde{W}^{a\mu\nu} - \frac{g_1^2}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

Want to connect anomaly to EW vacuum structure

Can write anomaly as total divergence:

$$\partial_\mu j^\mu_B = n_f \left(\frac{g^2}{32\pi^2} \partial_\mu K^\mu - \frac{g_1^2}{32\pi^2} \partial_\mu k^\mu \right)$$

$$K^\mu = \varepsilon^{\mu\nu\alpha\beta} \left(W^a_{\nu\alpha} A^a_\beta - \frac{1}{3} g \varepsilon_{abc} A^a_\nu A^b_\alpha A^c_\beta \right) \qquad k^\mu = \varepsilon^{\mu\nu\alpha\beta} F_{\nu\alpha} B_\beta$$

Consider path starting and ending in vacuum states...

$$\Delta B = \left[\int d^3x j^0_B \right]_{t=0}^{t_0} = n_f \frac{g_2^2}{32\pi^2} \left[\int d^3x K^0 \right]_{t=0}^{t_0}$$

$$\Delta B = n_f \Delta N_{CS}$$

Transitions between vacua with distinct winding numbers violate baryon number!

$$\Delta B \neq 0$$

Can we go *through* the barrier?

In QCD (SU(3)) with similar vacuum structure, we found classical solutions of *Euclidean* action interpolating between vacua ('instantons') with amplitude $\sim \exp(-S_E^{\text{instanton}})$

Can we generate baryons by quantum tunneling between vacua?

(Only option at zero temp)

Consider pure SU(2) instanton starting and ending in vacuum...

$$\int d^4x (W^a_{\mu\nu} - \tilde{W}^a_{\mu\nu})^2 = 4S_E - 2\left(\frac{16\pi^2}{g^2}\right) N_{CS} \geq 0$$

$$\Rightarrow \quad S_E \geq \frac{8\pi^2}{g^2} N_{CS}$$

$$\Gamma_{\text{inst}} \sim \exp(-2S_E) \sim 10^{-170}$$

Not one B violating event in Hubble volume in entire history of universe!!

Can we go *over* the barrier?

Early universe was at finite temp- classical transitions allowed from 'kicks' by thermal excitation above a given threshold...

What is this threshold?

Instead of one peak, vacua separated by *ridge* of barrier configurations



Lowest-energy point on ridge is saddle point (1 neg. eigenvalue)- unstable equilibrium with corresponding *static*, *classical* solution of action

This is 'sphaleron' whose energy represents the threshold for thermal activation to hop over barrier

What do sphalerons tell us about the transition rate?

Consider ideal pendulum $L = \frac{1}{2} m\ell^2\dot{\theta}^2 - mg\ell(1-\cos\theta)$

Periodic vacuum structure (periodic wave fxns) with $\theta_n = 2n\pi,\ n \in \mathbb{Z}$

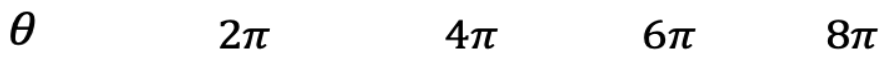
If $E_{\text{pend}} \ll E_{\text{bar}}$, can expand about single minimum ($\theta = 0$) and Schrodinger eq. becomes SHO (tunneling suppressed)

What if we couple to thermal bath?

Now pendulum will make spontaneous jumps over barrier.

'Sphaleron' is pendulum held static at $\theta = \pi$- energy of this config. (GPE of barrier) determines transition rate as

$$\Gamma(T) \sim \exp\left(-\frac{E_{\text{bar}}}{T}\right)$$



A Bit of Math... The Electroweak Sphaleron

Want to find static solution of classical field eqs. minimizing energy

$$E = \int d^3x \left[\frac{1}{4} W_{ij}^a W_{ij}^a + \frac{1}{4} F_{ij} F_{ij} + (D_i \phi)^\dagger (D_i \phi) + V(\phi) \right]$$

How do we find explicit expression for fields?

Build ansatz by modulating BCs by spatially-varying function

$$A_i^a \sigma^a dx^i = -\frac{2i}{g} f(gvr) dU^\infty (U^\infty)^{-1} \qquad \phi = \frac{v}{\sqrt{2}} h(gvr) U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\xi \equiv gvr \qquad U^\infty = \frac{1}{r} \begin{pmatrix} z & x+iy \\ -x+iy & z \end{pmatrix}$$

$$E = \frac{4\pi v}{g} \int_0^\infty d\xi \left[4 \left(\frac{df}{d\xi} \right)^2 + \frac{8}{\xi^2} [f(1-f)]^2 + \frac{1}{2} \xi^2 \left(\frac{dh}{d\xi} \right)^2 + [h(1-f)]^2 + \frac{1}{4} \left(\frac{\lambda}{g^2} \right) \xi^2 (h^2-1)^2 \right]$$

This is still messy... what are f,h??

We have ansatz... $A_i^a \sigma^a dx^i = -\frac{2i}{g} f(gvr) dU^\infty (U^\infty)^{-1}$ $\phi = \frac{v}{\sqrt{2}} h(gvr) U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

What is the boundary behavior we expect for f,h?

1. **Regularity at origin requires** $\lim_{\xi \to 0} f(\xi) = \alpha \xi^2$ $\lim_{\xi \to 0} h(\xi) = \beta \xi$

2. **At (spatial) infinity, f,h should approach 1 (vacuum) with deviation that decays with screening factor**

$$\lim_{\xi \to \infty} f = 1 - \gamma \exp(-M_W \cdot r) \qquad \lim_{\xi \to \infty} h = 1 - \frac{\delta}{\xi} \exp(-M_H \cdot r)$$

Strategy is to say f,h resemble origin BC behavior until radial cutoffs when switch to infinity BC behavior
Minimizing energy wrt cutoff parameters, can approximate

$$E_{\text{sph}} \approx 1.5 \frac{4\pi v}{g} \approx 9.1 \text{ TeV}$$

How do we know this configuration is the sphaleron?

The object we found is a *static*, *finite energy*, *classical* solution minimizing the EW energy functional (good hunch)

Manton '83 constructed explicit loop starting and ending in vacuum at $\mu = 0, \pi$ and passing through our ansatz at $\mu = \pi/2$

$$E^{\text{loop}}(\mu) = \frac{4\pi v}{g}(1.28\sin^2\mu + 0.24\sin^4\mu)$$

Reaches max at our configuration!

Can also calculate change in baryon number from vacuum to sphaleron

$$\Delta B = \frac{g^2}{32\pi^2}\int d^3x K^0\Big|_0^{t_f} = \frac{1}{2}$$

Sits 'halfway' between vacua!

$$\Delta B = \frac{1}{2}$$

Do sphalerons solve baryogenesis?

Our analysis presents a problem...

Rate of thermal 'kicks' over barrier

$$\Gamma_{\rm sph} \sim \exp\left(-\frac{E_{\rm sph}}{T}\right)$$

N_H 1 2 3 4

But this is rate for *both* $\Delta N_H > 0$ (baryon excess) *and* $\Delta N_H < 0$ (antibaryon excess)

Thermal kicks have no directional preference for transitions between degenerate vacua

Sphaleron events cannot generate *net* $\Delta B \neq 0$ on their own

We are implicitly running into 'Sakharov conditions' required for complete theory of baryogenesis:

1. B Violation (self-evident, satisfied by sphaleron events)
2. C, CP Violation (required for asymmetry in rates of $\Delta B > 0, \Delta B < 0$)
3. Departure from thermal equilibrium (equil. population of states only cares about *energy* of states)

EW Theory naturally has all three! (maximally C-violating, CKM phase violates CP, EWPT can escape equilibrium)

Universe has conspired to keep CKM phase $\ll 1$

EWPT is far too weak (continuous crossover)

How can we use sphalerons to build complete theory of baryogenesis?

One approach is to consider SM as EFT with new physics at UV scale M

Add higher-dim (CP-odd) operator $\hat{\mathcal{O}} = \frac{b}{M^2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$

Action now contains term $S \supset \int d^4x\hat{\mathcal{O}}(x) = \int d^4x\frac{b}{M^2}\mathrm{Tr}(\Phi^\dagger\Phi)\partial_\mu K^\mu$

$$= -\int d^4x\frac{b}{M^2}\partial_0\mathrm{Tr}(\Phi^\dagger\Phi)K^0 = \boxed{-\frac{b}{M^2}\int dtN_{CS}\int d^3x\frac{d}{dt}\mathrm{Tr}(\Phi^\dagger\Phi)}$$

Removes degeneracy of vacua, *biases* sphaleron events with $\Delta N_{CS} > 0$

Net production of baryons!!

Conclusions

1. Baryon asymmetry tied directly to EW vacuum structure
2. Finite temperature ‘sphaleron’ events are practical mechanism of B violation
3. For complete theory of EWBG, must combine sphalerons with BSM extensions

Ask me about using sphalerons for...

EWBG with 2 Higgs Doublets (e.g. MSSM)

Baryogenesis with neutrino mass models (Leptogenesis)

Baryogenesis with axion models (Axiogenesis)